УДК 330.115

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ANALYSIS OF ECONOMIC DATA IN CONDITIONS OF INTERVAL UNCERTAINTY

When analyzing economic data, quite often one has to deal with a situation where the values of the studied quantities are not exactly known, but it is possible to indicate the boundaries within which these quantities change, that is, there is ambiguity in the data. In this case, the problem under consideration belongs to the class of problems under conditions of interval uncertainty. In other words, when formalizing an economic problem for subsequent modeling, the researcher does not have information about the exact value of the required value, but knows the boundaries of the interval to which this value presumably belongs. For example, when analyzing the economic situation, especially if the situation is unstable, the analyst is forced to operate with an interval of product prices, the value of which depends on the exchange rate of the national currency.

Interval modeling and analysis of interval data was mainly used in tasks with non-statistically specified uncertainty. Therefore, the development of this method took place independently of the theory of probability and mathematical statistics. However, in recent years there has been a rapid development of methods of applied mathematical statistics in problems where statistical data are not numbers, but intervals. Such methods, called mathematical statistics of interval data, are promising in the study of observational results with superimposed errors [1].

It should be noted that one of the shortcomings of interval analysis, which is often subjected to fair criticism, is the lack of reasonable considerations on how the boundaries of the intervals under consideration are determined. Most often, this question remains behind the “frame” of research, and when solving applied problems, the boundaries of the intervals are considered a priori given.

At the same time, when solving a number of applied problems, it is possible to indicate the boundaries of the intervals of the quantities under consideration, relying on the theory of probability, and then apply operations on the interval quantities.

Suppose that in the mathematical model, based on which a certain problem is formulated, the probability p of some random event E appears. Most often, the value p can be considered only as a mathematical abstraction, which is unknown when solving an applied problem. Therefore, in practice, one has to operate with its assessment (frequency of the event)

$$p^* = \frac{n_E}{n},$$

where n_E is the number of trials favorable to the event E in the sample of n observations.

It is obvious that the replacement of an unknown probability p by its estimate p^* is justified only for a sufficiently large volume of n observations. At the same time, it is known from probability theory that for any frequency value p^* it is possible to construct a confidence interval, to which the unknown value p belongs with a confidence probability β.

This interval can be written in the form center-radius

$$I = \left\langle p^c, r \right\rangle,$$

where

$$p^c = \frac{p^* + t_\beta^2 / 2n}{1 + t_\beta^2 / n}, \qquad r = \frac{t_\beta \sqrt{\dfrac{p^*(1-p^*)}{n} + \dfrac{t_\beta^2}{4n^2}}}{1 + t_\beta^2 / n}.$$

Here,

$t_\beta = \arg\Phi^*\left(\frac{1+\beta}{2}\right) > 0$, $\Phi^*(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{\tau^2}{2}} d\tau$ – normal distribution function.

Using the example of solving the problem of expert assessment of the market capacity, we will show the methodology for the practical use of the considered ratios.

The effectiveness of management decisions largely depends on the correct assessment of the market situation. For such an assessment, the market is usually divided into m segments, each of which has potential consumers of this product. Considering that the behavior of buyers within each segment can vary significantly, they must be evaluated separately.

To assess the market capacity, a point probabilistic model is used

$$C = \sum_{i=1}^{n}\sum_{j=1}^{m} S_i P_{ij} L_j \ ,$$

where C is the oriented total market capacity for the entire group of goods; L_j – the number of enterprises in the j-th segment consuming the i-th product; S_i - the cost of the i-th product; P_{ij} is the probability that the i-th product will be in demand in the market in j-th segment, $\sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij} = 1$.

With unknowns P_{ij}, based on such a model, a constructive algorithm can be developed if we move from a point probability P_{ij} to its confidence interval [2]. To do this, first, in each j-th market segment, we randomly select a certain part of potential consumers, among whom we will conduct an expert survey about their consent to purchase the i-th product. As a result, the purchase frequency of the i-th product in the j-th segment can be estimated by the formula

$$P_{ij}^* = \frac{Q_{ij}}{b_j},\ i = \overline{1,n},\ j = \overline{1,m}\ ,$$

where b_j is the total number of surveyed enterprises in the j-th market segment, and Q_{ij} is the number of surveyed enterprises in the j-th market segment that agree to buy the i-th product.

This makes it possible to move from the point model to its interval counterpart

$$\mathrm{C} = \sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{S}_i \mathbf{I}_{ij} L_j \ ,$$

where $\mathrm{S}_i = \left\langle S_i^c, r_i^S \right\rangle$ is the interval to which the cost S_i of some i-th good belongs; L_j – the number of potential buyers of the i-th product in the j-th segment; $\mathbf{I}_{ij} = \left\langle P_{ij}^c, r_{ij} \right\rangle$ is the confidence interval to which the probability P_{ij} of purchasing the i-th product in the j-th segment belongs with probability β.

Further, using interval arithmetic operations, it is possible to investigate and determine the marginal demand for a given product. Also to compare the capacity of several products and make the best management decision.

Using this approach, it is possible to build a decision-making scheme by a team of independent experts. Such models will be constructive, since not point values of probabilities, but their confidence intervals will be used to make an agreed decision [3, p.5]. The use of interval data analysis methods gave a positive result when building a creditscoring model based on Bayesian strategies [4].

References

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